

# Lens Masses and Distances from Microlens Parallax and Flux

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## ABSTRACT

I present a novel method for measuring lens masses for microlensing events. By combining a measured lens flux with the microlens parallax  $\vec{\pi}_E$ , it is possible to derive the mass of the lens system without knowing the angular size of the Einstein ring,  $\theta_E$ . This enables mass and distance measurements for single, luminous lenses, as well as binary and planetary lenses without caustic crossings. I discuss applications of this method in the contexts of the *Spitzer*, *Kepler*, and *WFIRST* microlensing missions.

*Subject headings:* gravitational lensing: micro

## 1. Introduction

Microlensing is a powerful technique because it can probe a wide variety of systems including isolated black holes (e.g. Poindexter et al. 2005) and planetary systems distant from the Earth (e.g. Udalski et al. 2015). It probes lens systems regardless of their light because it uses light from a background source to create the signal. The flip side of this is that it is very difficult to determine the absolute properties of the lens system. For example, MOA-2011-BLG-262 has a 2-body lens with a planetary mass ratio  $q \sim 5 \times 10^{-4}$ , but the solution is ambiguous between an M-dwarf with a gas giant planet and a free-floating planet with a well separated sub-Earth mass moon (Bennett et al. 2014).

The reason it is so difficult to determine the properties of the lens is that its mass  $M_L$ , distance  $D_L$ , and motion relative to the source  $\mu_{\text{rel}}$  are all encoded in the primary microlens observable

$$t_E = \frac{\theta_E}{\mu_{\text{rel}}}, \quad (1)$$

where

$$\theta_E = \sqrt{\kappa M_L \pi_{\text{rel}}} \quad \text{where} \quad \pi_{\text{rel}} = \text{AU} \left( \frac{D_L - D_S}{D_S D_L} \right) \quad \text{and} \quad \kappa = 8.14 \frac{\text{mas}}{M_\odot}. \quad (2)$$

$D_S$  is the distance to the source, which is assumed to be in the Bulge (i.e. at  $\sim 8$  kpc). Hence, two additional measurements are needed to determine the lens mass and distance.

Historically, measurements of the lens mass have focused on combining a measurement for the angular size of the Einstein ring,  $\theta_E$ , with other information.  $\theta_E$  is often measured for 2-body

microlensing events. This is because 2-body microlenses are identified as such by interactions of the source light with the lens caustic structure. The caustic is a closed curve at which the magnification formally diverges to infinity. As a result, if the source crosses a caustic, the observed magnification depends sensitively on the physical size of the source, which may be expressed as  $\rho = \theta_\star/\theta_E$  where  $\theta_\star$  is the angular size of the source. Hence, if the lens is observed to have two bodies, it is likely that a caustic crossing has been observed and thus, that  $\rho$  is measured, which leads to a measurement of  $\theta_E$  once  $\theta_\star$  is determined from the source color and magnitude (Yoo et al. 2004).

Once  $\theta_E$  is measured, this yields a mass-distance relationship for the lens star such as those shown by the blue lines in Figures 1 and 2. If this can be combined with a second mass-distance relationship, then the mass and distance to the lens are determined. This may be done in one of two ways. First, if parallax effects are measured in the light curve, the measurement of the microlens parallax vector  $\vec{\pi}_E$  directly gives the lens mass and distance:

$$M_L = \frac{\theta_E}{\kappa\pi_E}; \quad D_L = (\pi_E\theta_E - \pi_S)^{-1}, \quad (3)$$

where  $\pi_S = D_S^{-1}$ . The second method is to make a measurement of the lens flux using high resolution imaging so that stars unrelated to the microlensing event are resolved. This may be done either while the lens and source are still superposed or after they have separated, depending on the brightness of the source. This flux measurement then gives a magnitude-distance relationship that can be compared to the mass-distance relationship from  $\theta_E$  via stellar isochrones, e.g. the magenta lines in Figure 1 and 2.

However, there are many cases in which  $\theta_E$  is not measured. This includes almost all point lenses, for which a caustic crossing requires that the lens transit the face of the source, and also 2-body systems without caustic crossings, a situation that is not uncommon for stellar binary lenses. For these systems, I propose an alternate means of determining the lens mass (and distance) from the parallax.

## 2. Masses from Parallax and Flux

In detail, the microlens parallax,  $\vec{\pi}_E$ , is measured from the apparent displacement as a function of time of the lens relative to the source due to the parallax effect as compared to what would be observed from rectilinear motion as seen from a single location. For example, two observers at different locations will see a different alignment for the source and lens, and therefore a different projected separation. In addition, as with all separations in microlensing, this displacement is measure relative to the size of the Einstein ring. Hence, the microlens parallax is a vector that depends on the lens-source relative parallax,  $\pi_{\text{rel}}$ , the direction of the lens-source relative proper motion,  $\vec{\mu}_{\text{rel}}/\mu_{\text{rel}}$ , and is scaled to the size of the Einstein ring,  $\theta_E$ :

$$\vec{\pi}_E = \frac{\pi_{\text{rel}}}{\theta_E} \frac{\vec{\mu}_{\text{rel}}}{\mu_{\text{rel}}}. \quad (4)$$

The basic idea of measuring masses from parallax and flux stems from Equation 3. If a

measurement of the lens flux plus a measurement of  $\theta_E$  can lead to a measurement of the lens mass, so too must a measurement of the lens flux plus a measurement of  $\vec{\pi}_E$  lead to a measurement of the lens mass. This is just the third possible pairing of the luminosity-distance relationships shown in Figures 1 and 2.

Previously, Dong et al. (2009) combined constraints from several microlensing effects, the strongest of which was parallax, with an *HST* measurement of the lens flux to determine the mass of OGLE-2005-BLG-071. However, there has not been a measurement of a lens mass from only parallax and lens flux.

The reasons this idea has never before been applied in this form are two-fold. First, most of the microlensing events that have been published and for which absolute (rather than statistical) masses are necessary have been 2-body lenses, especially star-planet systems. In those cases,  $\theta_E$  is almost always measured, so the techniques described in Section 1 are sufficient. Second, before the advent of space-based microlensing, microlens parallaxes were rarely measured, so the most frequent situation was one with a measurement of  $\theta_E$  but no measurement of  $\vec{\pi}_E$  (the first space-based parallax measurement was made by Dong et al. 2007).

However, the *Spitzer* microlens parallax campaign (and soon the *K2* microlensing campaign) has enabled highly precise measurements of the microlens parallax for a significant number of events. In fact, in the case of planetary microlens OGLE-2014-BLG-0124L, the limiting factor in determining the lens mass was the precision of  $\theta_E$  rather than any uncertainty in the microlens parallax. In addition, *WFIRST* will measure at least one component of the parallax vector extremely precisely for large numbers of events as well as measuring lens fluxes, so the combination of those two pieces of information will yield more detail about the nature of the lens. In the next section, I give additional details about how this idea can be applied to these space missions.

### 3. Specific Applications

#### 3.1. Satellite Parallaxes

Microlens parallax measurements from simultaneous observations from the Earth and a satellite in solar orbit (such as *Spitzer* or *Kepler*), can yield very precise measurements of the parallax (Calchi Novati et al. 2015; Udalski et al. 2015; Yee et al. 2015; Zhu et al. 2015), albeit subject to the four-fold degeneracy (Refsdal 1966; Gould 1994). If this degeneracy can be resolved, the resulting mass-distance relationship is well-defined (e.g. black lines in Figure 1).

The four-fold degeneracy may be resolved in several ways. First, note that the four-fold degeneracy arises from a two-fold degeneracy in magnitude and a two-fold degeneracy in direction. Of these, only the degeneracy in magnitude affects the measured lens mass and distance. For high-magnification microlensing events, the difference in the magnitude of  $\vec{\pi}_E$  for the different solutions is small enough that it may be ignored (Gould & Yee 2012). In addition, higher-order effects in the

light curve can break the degeneracy outright. Alternatively, Calchi Novati et al. (2015) showed that the degeneracy may be broken in some cases using the Rich argument. This is a statistical argument that if the two components of the parallax vector are highly unequal in one case and close to equal in the other, the latter is more likely (see Calchi Novati et al. 2015, for a full explanation). Finally, other ideas exist for breaking this degeneracy in specific circumstances (e.g. Gould 2013; Yee 2013, 2015).

Figure 1 provides a specific example from the 2014 *Spitzer* microlens parallax campaign. Udalski et al. (2015) measured the parallax for this event, but finite source effects were not observed. However, they were able to place an upper limit on  $\rho$  based on the fact that a larger source would have led to observable effects on the light curve, and they were able to set a lower limit on  $\rho$  from an upper limit on the lens flux. Based on these constraints, they estimated the mass and distance of this planetary system.

Using the 4.0 Gyr, solar metallicity isochrone from An et al. (2007) and assuming the source is at 8 kpc, I transform the two mass-distance relations from parallax and finite source effects into absolute magnitude-distance relations as shown in the figure. Note that the relation based on finite source effects is derived from the nominal value and uncertainty in  $\rho$  quoted in Table 1 of Udalski et al. (2015). However, their analysis finds that  $\rho$  is more or less uniformly distributed between the constraints described above. I also construct the absolute magnitude-distance relation that would result from a direct measurement of the lens flux, assuming a  $0.71M_{\odot}$  lens at 4.1 kpc and that the extinction varies linearly with distance and has a total value  $A_H = 0.399$  (Schlafly & Finkbeiner 2011). The uncertainty in this relation assumes that the source and lens are still superposed, so that the lens flux must be derived from a measurement of their combined flux, and that the uncertainty in this flux measurement is 0.05 mag. Note that this figure shows just one of the two solutions from Udalski et al. (2015), but the other solution is almost identical.

Because of the uncertainty in  $\rho$ , a flux measurement of OGLE-2014-BLG-0124L, combined with the known parallax, would yield a direct measurement of the lens (and planet) mass in this system.

In addition, most microlenses from the *Spitzer* sample have parallax measurements but no measurement of finite source effects because they are single objects. For these objects, Calchi Novati et al. (2015) were only able to make statistical estimates of the lens distances from the measured parallaxes through a kinematic argument that assumes  $\mu_{\text{rel}}$  is approximately known based on the idea that the Sun and the lens are moving together. Hence, a flux measurement is the only way to provide direct measures of the lens masses and distances. One specific example is OGLE-2015-BLG-1285, for which a flux measurement of the lens secondary would lead to a mass measurement for the system and therefore determine whether the primary is a black hole or a neutron star (Shvartzvald et al. 2015). Moreover, direct measurements of the lens fluxes would improve the measurements of their distances, which would in turn improve the measurement of the Galactic distribution of planets.

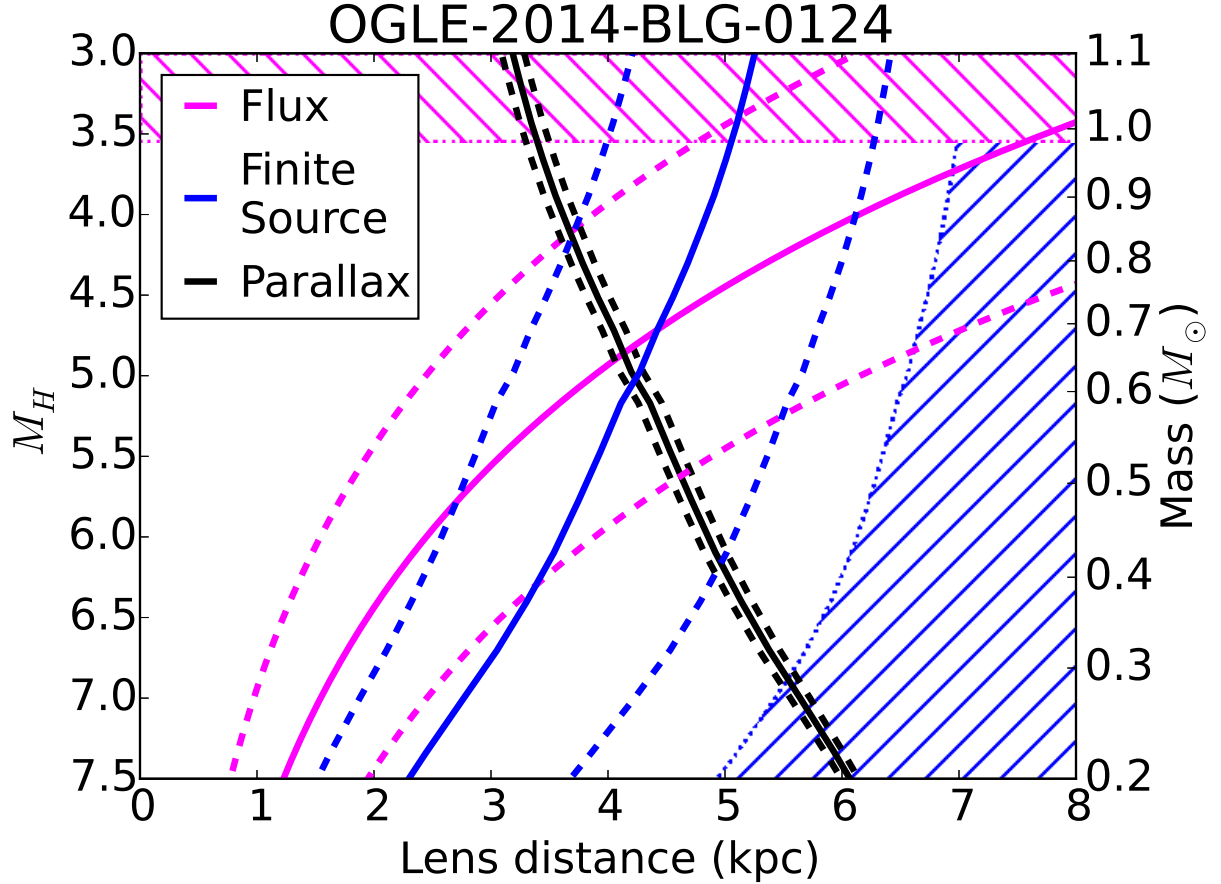


Fig. 1.— Absolute Magnitude and Mass constraints on OGLE-2014-BLG-0124L, a planetary system from the 2014 *Spitzer* campaign. For this microlensing event, the parallax measured from the difference in the light curves as seen from *Spitzer* and the Earth is extremely precise, yielding the mass-distance relation shown in black. However, the source size was not well measured from finite source effects, leading to a large uncertainty in the mass of the system. The light curve itself yields only an upper limit on the source size, which excludes the blue hatched region, while upper limits on the lens flux exclude the magenta hatched region. The blue curves show the mass-distance relation from the nominal value of  $\rho$  from Udalski et al. (2015), but anything in the non-excluded region is consistent with the light curve at the 3- $\sigma$  level. The magenta line shows the hypothetical constraints from a measurement of the lens flux assuming that the lens star is an  $0.71M_\odot$  star located at 4.1 kpc as in the Udalski et al. (2015) solution. 1- $\sigma$  uncertainties are shown as the dashed lines.

### 3.2. WFIRST

This method of combining lens flux and parallax measurements is also relevant for measuring lens masses and distances with *WFIRST*. However, in this case, there is more information available. *WFIRST* will make three measurements that constrain the lens mass. First, because of its higher resolution, the microlensing observations will resolve out blended background stars. Hence, any light left over will be due to the lens, a companion to the lens, or a companion to the source. In general, the relative probabilities of these various scenarios can be calculated, so lens flux measurements will be routine. Second, the precision of *WFIRST* will allow the measurement of astrometric microlensing effects, which gives a measurement of  $\theta_E$  (Gould & Yee 2014). Finally, *WFIRST* will measure parallaxes from the orbital motion of the satellite about the Sun. Because the events are short, orbital parallax measurements are primarily sensitive to the parallel component of the microlens parallax vector  $\pi_{E,\parallel} = \vec{\pi}_E \sin \lambda$ , where  $\lambda$  is the latitude of the event with respect to the ecliptic (Gould 2013). This leads to a 1-D measurement of the parallax (Gould et al. 1994), a problem which is exacerbated by the fact that the ecliptic runs through the Galactic Bulge. However, if the parallax is measured better, e.g. because the parallax is large or more complex parallax effects are observed (cf. Gould 2013; Yee 2013), this measurement of the parallax is quite powerful because it takes a completely different form from the other relations.

Figure 2 illustrates the interplay between these three measurements for a typical case of a  $0.5M_\odot$  lens star at 4.0 kpc and a case in which the lens is much closer ( $D_L = 1.0$  kpc). I have assumed the source is a dwarf star at 8 kpc with  $H = 18.0$  mag, known with a precision of 0.05 mag from the microlensing model. For the purposes of measuring the flux of the lens, I have adopted an uncertainty in the calibrated flux at baseline of 0.05 mag and assumed linearly varying extinction with a total value of  $A_H = 0.4$ . For the measurement of  $\theta_E$  from microlens astrometry, I have used Equation 18 from Gould & Yee (2014) and adopted their fiducial parameters (i.e.,  $\sigma_{\text{phot}} = 0.01$ , FWHM = 175 mas,  $N = 7000$ , and  $\beta = 0.7$ ). Finally, for the parallax, I show two cases. The hatched regions show the region excluded if only 1-D parallaxes are measured (with  $\lambda = 30^\circ$ ), while the dashed lines assume a 10% uncertainty in the total magnitude of the parallax vector.

This figure clearly shows that what can be learned from the combination of lens flux and parallax depends on how well the parallax is measured and somewhat on the orientation of  $\vec{\pi}_E$  (as  $\lambda \rightarrow 90^\circ$ ,  $\pi_{E,\parallel} \rightarrow \pi_E$ , so the constraints from 1-D parallaxes improve). However, the subset of cases for which the parallax is measured are important for validating the *WFIRST* results.

The most striking thing about Figure 2 is that the luminosity-distance relationship derived from the parallax has a completely different form from that derived from  $\theta_E$ . This provides an important consistency check for the derived mass, especially as the relations from  $\theta_E$  and flux become degenerate for low-mass, nearby lenses (see left panel of Figure 2). This consistency check is especially important for  $\theta_E$  measurements derived from microlens astrometry. The *WFIRST* microlensing mission will be the first time the astrometric microlensing effect will be measured for large numbers of events. Hence, the systematics inherent in this method are completely unknown.

The independent prediction of the lens mass from its measured flux and parallax will provide a means to test the astrometric measurements and vet for any systematics.

#### 4. Summary

I have shown how microlens parallax measurements may be combined with a measurement of the lens flux to yield a measurement of the lens mass and distance. This method is particularly important for measuring masses of lenses without measurements of  $\theta_E$ , a category which includes almost all single lenses. It is also relevant for low-mass, nearby lenses for which the mass-distance relations derived from flux and from  $\theta_E$  are partially degenerate.

Although in the past, many events of interest were more likely to have  $\theta_E$  measurements as opposed to  $\pi_E$  measurements, that situation is changing. First, that bias was partly a selection effect in the publication record. Second, the *Spitzer* and *K2* microlensing campaigns are enabling highly precise parallax measurements for microlensing events. In addition, higher cadence, higher precision microlensing surveys allow the detection of more subtle planetary signals, so measurements of  $\theta_E$  are no longer guaranteed for those events (Zhu et al. 2014). For these systems, combining parallax and flux measurements is the only way to measure the lens mass. Finally, these issues will become even more acute for the *WFIRST* microlensing mission, which will routinely measure parallaxes and lens fluxes for a significant fraction of microlensing events.

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#### REFERENCES

- An, D., Terndrup, D. M., Pinsonneault, M. H., et al. 2007, ApJ, 655, 233
- Bennett, D. P., Batista, V., Bond, I. A., et al. 2014, ApJ, 785, 155
- Calchi Novati, S., Gould, A., Udalski, A., et al. 2015, ApJ, 804, 20
- Dong, S., Udalski, A., Gould, A., et al. 2007, ApJ, 664, 862
- Dong, S., Gould, A., Udalski, A., et al. 2009, ApJ, 695, 970
- Gould, A. 1994, ApJ, 421, L71
- . 2013, ApJ, 763, L35

- Gould, A., Miralda-Escude, J., & Bahcall, J. N. 1994, *ApJ*, 423, L105
- Gould, A., & Yee, J. C. 2012, *ApJ*, 755, L17
- . 2014, *ApJ*, 784, 64
- Poindexter, S., Afonso, C., Bennett, D. P., et al. 2005, *ApJ*, 633, 914
- Refsdal, S. 1966, *MNRAS*, 134, 315
- Schlafly, E. F., & Finkbeiner, D. P. 2011, *ApJ*, 737, 103
- Shvartzvald, Y., Udalski, A., Gould, A., et al. 2015, *ArXiv e-prints*, arXiv:1508.06636
- Udalski, A., Yee, J. C., Gould, A., et al. 2015, *ApJ*, 799, 237
- Yee, J. C. 2013, *ApJ*, 770, L31
- Yee, J. C., Udalski, A., Calchi Novati, S., et al. 2015, *ApJ*, 802, 76
- Yee, J. C. 2015, in prep
- Yoo, J., DePoy, D. L., Gal-Yam, A., et al. 2004, *ApJ*, 616, 1204
- Zhu, W., Gould, A., Penny, M., Mao, S., & Gendron, R. 2014, *ApJ*, 794, 53
- Zhu, W., Udalski, A., Gould, A., et al. 2015, *ApJ*, 805, 8



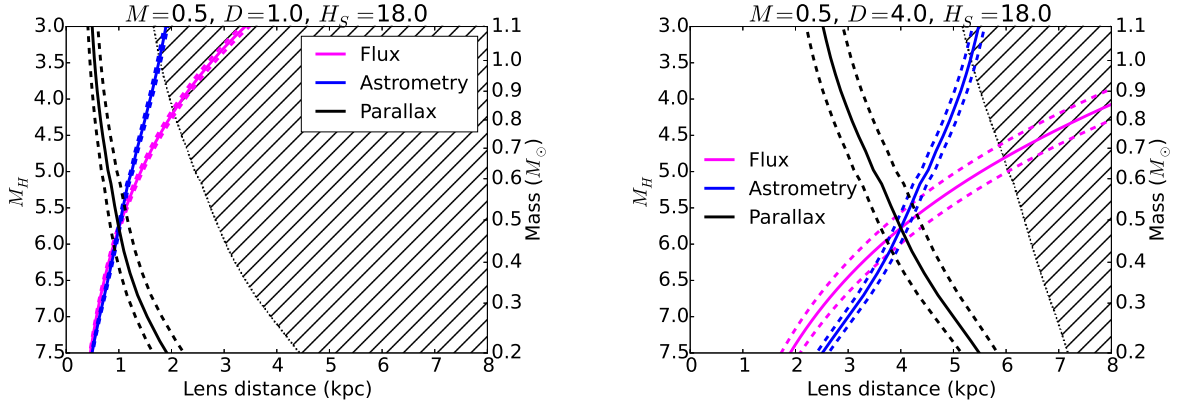


Fig. 2.— Absolute Magnitude-Distance relations for a  $0.5M_{\odot}$  star at 1.0 kpc (left) and 4.0 kpc (right). *WFIRST* will measure three different constraints: the flux of the lens (magenta), astrometric microlensing (blue), and parallax (black). For parallaxes, *WFIRST* will be much more sensitive to the parallel component  $\pi_{E,\parallel}$  than to the perpendicular component  $\pi_{E,\perp}$ . The hatched region shows the region that is ruled out if only  $\pi_{E,\parallel}$  is measured (assuming  $\arctan \pi_{E,\parallel}/\pi_{E,\perp} = 30^\circ$ ); the dashed lines show the 1- $\sigma$  uncertainties if the parallax is measured to 10%.